Government of Western Australia
School Curriculum and Standards Authority

MATHEMATICS SPECIALIST

Unit 1 and Unit 2

Formula Sheet

(*For use with Year 11 examinations and response tasks*)

2015/75004v6

This document is valid for teaching and examining from 1 July 2015.

Measurement

Circle:	$C = 2\pi r = \pi D$, where C is the circumference, r is the radius and D is the diameter $A = \pi r^2$, where A is the area
Triangle:	$A = \frac{1}{2} bh$, where b is the base and h is the perpendicular height
Parallelogram:	$A = bh$
Trapezium:	$A = \frac{1}{2}(a + b)h$, where a and b are the lengths of the parallel sides
Prism:	$V = Ah$, where V is the volume and A is the area of the base
Pyramid:	$V = \frac{1}{3} Ah$
Cylinder:	$S = 2\pi rh + 2\pi r^2$, where S is the total surface area $V = \pi r^2 h$
Cone:	$S = \pi rs + \pi r^2$, where s is the slant height $V = \frac{1}{3}\pi r^2 h$
Sphere:	$S = 4\pi r^2$ $V = \frac{4}{3}\pi r^3$

Combinatorics

Combinations

Number of arrangements: (of n different objects in an ordered list)

$$n(n-1)(n-2)\times.....\times3\times2\times1=n!$$

Number of combinations: (of r objects taken from a set of n distinct objects)

$$\binom{n}{r}=\frac{n!}{r!(n-r)!}; \qquad \binom{n}{r}=\binom{n}{n-r}; \qquad \binom{n}{0}=1$$

Number of permutations: (of r objects taken from a set of n distinct objects)

$$^nP_r=n(n-1)(n-2)...(n-r+1)=\frac{n!}{(n-r)!}$$

Number of permutations with some identical objects: $\quad ^nP_r=\dfrac{n!}{r_1!r_2!r_3!...}$

Inclusion – exclusion principle: $\quad |A\cup B|=|A|+|B|-|A\cap B|$

$$|A\cup B\cup C| = |A| + |B| + |C| - |A\cap B| - |A\cap C| - |B\cap C| + |A\cap B\cap C|$$

Vectors in the Plane

Representing vectors

Magnitude of a vector: $\quad |\mathbf{a}| = |(a_1,a_2)| = \sqrt{a_1^2+a_2^2}$

Algebra of vectors

Unit vector: $\quad \hat{\mathbf{a}}=\dfrac{\mathbf{a}}{|\mathbf{a}|}$

Scalar product: $\quad \mathbf{a}\bullet\mathbf{b} = |\mathbf{a}||\mathbf{b}|\cos\theta \quad$ or $\quad \mathbf{a}\bullet\mathbf{b} = a_1b_1+a_2b_2$

Vector projection (of **a** on **b**): $\quad \mathbf{p} = (\mathbf{a}\bullet\hat{\mathbf{b}})\hat{\mathbf{b}} = |\mathbf{a}|\cos\theta\ \hat{\mathbf{b}}$

Trigonometry

Basic trigonometric functions

$\sin(-\theta) = -\sin\theta$ $\cos(-\theta) = \cos\theta$ $\tan(-\theta) = -\tan\theta$

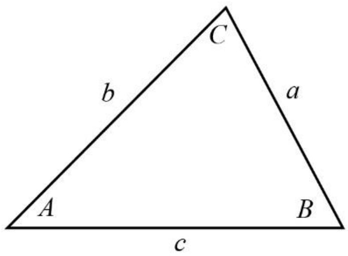
$\sin\left(\theta + \frac{\pi}{2}\right) = \cos\theta$ $\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$

Cosine and sine rules

For any triangle ABC with correspondimg length of sides a,b,c

Cosine rule: $c^2 = a^2 + b^2 - 2ab\cos C$

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$



Area of Δ :

$$A = \frac{1}{2}ab\sin C$$

$$= \sqrt{s(s-a)(s-b)(s-c)} \text{ where } s = \frac{1}{2}(a+b+c)$$

Circular measure and radian measure

In a circle of radius r for an arc subtending angle θ (radians) at the centre

Length of arc: $\ell = r\theta$ Length of chord: $l = 2r\sin\frac{1}{2}\theta$

Area of sector: $A = \frac{1}{2}r^2\theta$ Area of segments: $A = \frac{1}{2}r^2(\theta - \sin\theta)$

Compound angles

Angle sum and difference identites:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

Double angle identities:

$$\sin 2A = 2\sin A\cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Reciprocal trigonometric functions

$$\sec\theta = \frac{1}{\cos\theta},\ \cos\theta \neq 0 \qquad \operatorname{cosec}\theta = \frac{1}{\sin\theta},\ \sin\theta \neq 0 \qquad \cot\theta = \frac{1}{\tan\theta},\ \tan\theta \neq 0$$

Trigonometric identities

Pythagorean identities: $\sin^2\theta + \cos^2\theta = 1 \qquad 1 + \tan^2\theta = \sec^2\theta \qquad \cot^2\theta + 1 = \operatorname{cosec}^2\theta$

Product identities:

$$\cos A\cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$\sin A\sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\sin A\cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

$$\cos A\sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

Auxilary angle formulae:

$a\sin x \pm b\cos x = R\sin(x \pm \alpha)$ for $0 < \alpha < \frac{\pi}{2}$, where $R^2 = a^2 + b^2, \tan\alpha = \frac{b}{a}$

Triple angle identities:

$$\sin(3A) = 3\sin A - 4\sin^3 A$$

$$\cos(3A) = 4\cos^3 A - 3\cos A$$

$$\tan(3A) = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

Matrices

Matrix arithmetic

Identity matrix: If $\mathbf{A}$ is invertible, $\mathbf{AA}^{-1} = \mathbf{I}$ where $\mathbf{I}$ is the identity matrix

Inverse matrix: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Determinant: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det \mathbf{A} = ad - bc$

Transformation Matrices

Dilation: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$

Rotation: $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ where θ is an anti-clockwise rotation about the origin

Reflection: $\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ where the reflection is in the line $y = x\tan\theta$

Real and Complex numbers

Number Sets

Natural Numbers: $\mathbb{N} := \{1,2,3,\ldots..\}$
Integer Numbers: $\mathbb{Z} := \{\ldots -2,-1,0,1,2,\ldots\}$
Rational Numbers: $\mathbb{Q} := \left\{q : q = \frac{a}{b},\ \text{where a and b are integers},\ b \neq 0\right\}$
Irrational Numbers: Numbers that cannot be expressed as the ratio of two integers
Real Numbers: The set of all rational and irrational numbers ($\mathbb{R}$)
Complex Numbers: $\mathbb{C} := \{z : z = ai + b,\ \text{where } a,b \in \mathbb{R},\ i^2 = -1\}$

Complex Numbers

For $z = ai + b,\ \text{where } a,b \in \mathbb{R},\ i^2 = -1$

Modulus: $\operatorname{mod} z = |z| = |a+ib| = \sqrt{a^2+b^2}$

Product: $|z_1 z_2| = |z_1||z_2|$

Conjugate: $\bar{z} = a - ib$, $z\bar{z} = |z|^2$, $\overline{z_1 + z_2} = \bar{z}_1 + \bar{z}_2$, $\overline{z_1 z_2} = \bar{z}_1\bar{z}_2$

Other useful results

Binomial expansion: $(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{r} x^{n-r} y^r + \cdots + y^n$

Binomial coefficients: $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \times (n-1) \times \cdots \times (n-r+1)}{r \times (r-1) \times \cdots \times 2 \times 1}$

Index laws:

For $a, b > 0$ and m, n real,

$a^m b^m = (ab)^m$ $\quad a^m a^n = a^{m+n}$ $\quad (a^m)^n = a^{mn}$

$a^{-m} = \frac{1}{a^m}$ $\quad \frac{a^m}{a^n} = a^{m-n}$ $\quad a^0 = 1$

For $a > 0$, m an integer and n a positive integer, $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

Arithmetic sequences

For initial term a and common difference d:

$T_n = a + (n-1)d, n \geq 1$

$T_{n+1} = T_n + d$, where $T_1 = a$

$S_n = \frac{n}{2}\left(2a + (n-1)d\right)$

Geometric sequences

For initial term a and common difference r:

$T_{n+1} = rT_n$, where $T_1 = a$

$T_n = ar^{n-1}, n \geq 1$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}, \quad |r| < 1$

Lines and Linear relationships

For points $P(x_1, y_1)$ and $Q(x_2, y_2)$

Mid-point of P and Q: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Gradient of the line through P and Q: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Equation of the line through P with slope m: $y - y_1 = m(x - x_1)$

Parallel lines: $m_1 = m_2$

Perpendicular lines: $m_1 m_2 = -1$

General equation of a line: $ax + by + c = 0$ or $y = mx + c$

Quadratic relationships

For the general quadratic equation $ax^2 + bx + c = 0,\ a \neq 0$

Completing the square: $ax^2 + bx + c = a\left(x + \dfrac{b}{2a}\right)^2 + \left(c - \dfrac{b^2}{4a}\right)$

Discriminant: $\Delta = b^2 - 4ac$

Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Graphs and Relations

Equation of a circle: $(x-a)^2 + (y-b)^2 = r^2$

where, (a, b) is the centre and r is the radius

Note: Any additional formulas identified by the examination writers as necessary will be included in the body of the particular question.